Gravity coupling from micro-black holes

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Abstract: Recently much work has been done in lowering the Planck threshold of quantum gravitational effects (sub-millimeter dimension(s), Horava-Witten fifth dimension, strings or branes low energy effects, etc.). With a semi-classical calculation based on Hawking evaporation of planckian micro-black holes, I shall show in this paper as quantum gravity effects have to be present also at GUT energies.

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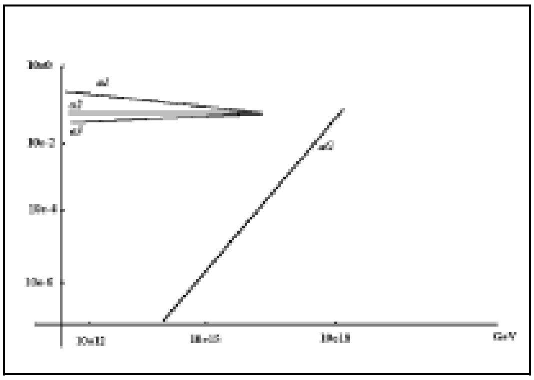
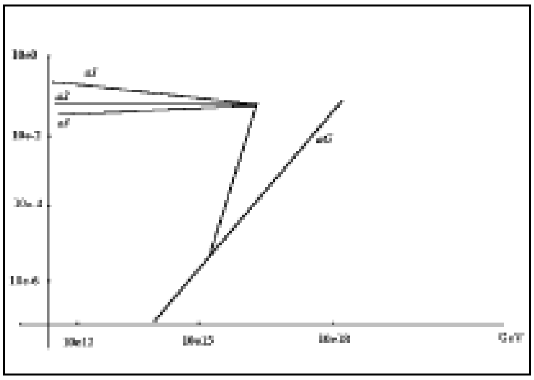
1. Introduction

As it is well known, the standard picture of supersymmetric GUT models predicts that the couplings of the three gauge interactions (electromagnetic, weak and color) unify to good accuracy at an energy around 2×10^{16} GeV. Gravity, on the contrary, presents another natural energy scale: quantum fluctuations of the gravitational field seem to become important only when we observe them at the Planck length $L_P = (G\hbar/c^3)^{1/2} = 1.6 \times 10^{-33}$ cm. Energy fluctuations of the order of $E_P = (\hbar c^5/G)^{1/2} = 1.2 \times 10^{19}$ GeV create at this scale microblack holes that modify the topology of spacetime. At the same scale, following the common views, gravity coupling (that is the"strenght" of gravitational interaction) should become comparable with those of the other three gauge interactions. This standard picture can be summarized in the figure 1.



During the last years, studies based on string theories have changed this vision[1]. In the early times of superstring theory, it was usual to associate it with (sub)planckian physics, because the theory provided an ultraviolet regulator of quantum gravity. This was particularly natural with weakly coupled heterotic string which gives a promising qualitative description of supersymmtric unification scenario at M_{GUT} . The simple tree-level expression for the string scale, $M_H \simeq gM_P$ (where M_P is the Planck mass and $g \simeq 1/5$ is the string coupling), valid in the context of perturbative heterotic string, enforced these beliefs. More recently, a number of authors have considered the possibility that the compactification energy scale is far lower, with a fundamental scale of string theory being as low as TeV. Since the gauge couplings are inversely proportional to the volume of the compatification space, this implies large compactification volumes and therefore large extra dimension(s). By modifying the compactification radius one can tune the couplings of gauge interactions, including gravity.



The heterotic string compactified on large volumes is strongly coupled and, by duality, it is well described by a weak coupled theory (type I or type II). A general property of these theories is that gauge interactions are localized on p-branes (with $p \leq 9$) while gravity propagates in different spacetime dimensions. Precisely, gauge fields and particles with gauge charges move only on the "walls" (i.e. on the p-branes) while gravity moves in the bulk, the spacetime region whose the p-branes are the boundary[2]. This is the Horava-Witten proposal: the branes represent 3+1 dimensional walls in which all the Standard Model particles live, while gravity moves in the 4+1 dimensional bulk between the walls. The other

six additional dimensions of string theory should be much smaller than that inhabited by gravity. The wall is then 9+1 dimensional in all and the complete spacetime is 10+1 dimensional. One can choose the size of the 11th dimension so that all four forces meet at the same common scale, presumably the GUT scale (see figure 2)[3] .
The basic idea is thus very simple: modifying the size of this extra dimension(s) where gravity propagates, the coupling of gravity can be modified as to raise the unification point with the other three interactions. And, very cleverly, the couplings of other interactions remain untouched by this extra dimension.

2. Gravity coupling from micro-black holes

The purpose of this section is to show how the threshold of quantum gravitational effects can be lowered from M_P to M_{GUT} without the use of large extra dimensions, of 11 dimensional M-Theory or of Horava-Witten model. Also in the framework of semiclassical 4 dimensional gravity a careful analysis brings to the conclusion that quantum gravity effects should be present at the GUT scale. This result surely enforces also string theorists' speculations because gives to them a valuable semiclassical (i.e. non-stringy) base. In other words, effects of this kind should be rather model independent and therefore should belong to real physics and not only to a particular theoretical framework of predictions.
Let us begin by reminding the process of creation of a micro black hole. We can use the Heisenberg inequality $\Delta p \Delta x \geq \hbar / 2$ casted in the form $\Delta E \Delta x \geq \hbar c / 2$ (beacause $\Delta E \sim c \Delta p$ in our high energy situation) to observe that in a space region of width Δx the metric field can fluctuate with an amplitude in energy of $\Delta E \sim \hbar c / 2 \Delta x$. If the gravitational radius

$R_g = \frac{2G\Delta E}{c^4}$ associated with the energy ΔE equals the width Δx of the space region, a micro-black hole originates[4] . This happens when $\Delta x = L_P$ and the typical energy of the process is of course the Planck energy E_P. Micro holes are therefore typical quantum gravitational objects. Now, let us suppose that the evaporation mechanism via Hawking radiation hold its validity also at the Planck scale, and therefore the micro hole evapotate just as its bigger brothers. We can calculate the lifetime of a black hole of given inital mass M_0 , loosing mass via Hawking radiation, in a strightforward manner.
Using the Stefan-Boltzmann law

$$W = \sigma T^4$$

we can say that the variation dM of the mass of the hole in a time dt is

$$dM = -\frac{\sigma T^4 S}{c^2} dt$$

where S is the surface of the hole and T is the Hawking temperature. For a Schwarzschild black hole we have

$$S = 4\pi R_g^2 \quad with \quad R_g = \frac{2GM}{c^2} \quad and \quad T = \frac{\hbar c^3}{8\pi kGM}$$

The equation in dM becomes

$$dM = -\frac{\lambda}{M^2} dt$$

with

$$\lambda = \frac{\sigma \hbar^4 c^6}{2^8 \pi^3 G^2 k^4}$$

This gives the mass of the hole at time t

$$M(t) = (M_O^3 - 3\lambda t)^{1/3}$$

and therefore the lifetime of the hole

$$t_o = \frac{M_O^3}{3\lambda} = \frac{2^8 \pi^3 G^2 k^4}{\sigma \hbar^4 c^6} \frac{M_O^3}{3}$$

In particular, for a micro-black hole of Planck mass, we obtain for the lifetime

$$t_o = \frac{2^5}{3} \frac{\pi^3 k^4}{\sigma} \frac{1}{\hbar^3 c^2} \left(\frac{G\hbar}{c^5}\right)^{1/2} \sim 2.01 \cdot 10^3 \tau_P$$

where $\tau_P = \left(\frac{G\hbar}{c^5}\right)^{1/2}$ is the Planck time.

We note that an energy fluctuation of the same size of a micro hole would have a lifetime of only one Planck time, if there weren't gravitational effects (i.e. the event horizon of the micro hole which traps the energy and the Hawking evaporation) and if we took into account the Heisenberg principle only. Thus we can consider the micro hole, which is a genuine quantum object, as a sort of *metastable quantum state*. From the theory of the decay of metastable[5] states we can infer that the decay probabilty dP during the time interval (t, t+dt) of the state is proportional to

$$\exp[-\frac{1}{\hbar}\ \Gamma\ t]$$

where Γ is the wideness in energy of the state. The mean lifetime of the state is

$$\tau = \frac{\hbar}{\Gamma}$$

From the lifetime of the quantum state *micro hole*, which is about 2000 τ_P , we can estimate the wideness in energy of the metastable micro-black hole state

$$\Gamma = \frac{\hbar}{2000\tau_P} = \frac{E_P}{2000} = \frac{1.2 \cdot 10^{19}}{2000} GeV = 6 \cdot 10^{15}\ GeV$$

This wideness, as one can immediately see, allows the existence of micro-black holes also at the GUT threshold, well below the Planck energy. And probably also at energies around 1-10 TeV.

3. Conclusion

The GUT energy scale emerges therefore in a natural way from a careful analysis of the micro-black hole metastable state. This seems to indicate that typical quantum gravity objects can be present also at energies well below the Planck threshold. And it seems to corroborate from a semiclassical point of view the string theory claim that the fundamental scale for unification may be the M_{GUT} scale. It seems that the quantum gravity scale can be fixed definitly at the GUT scale (and perhaps below) and no more at the Planck scale. This agrees also with the simplest inflationary potential invoked to explain the density fluctuations as measured by COBE, which goes as $M^4_{GUT}\ V(\Phi/M_P)$. (See Banks et al. [1]).

References

[1] See, for example, I. Antoniadis, On possible modifications of gravitation in the (sub)millimeter range, LANL hep-ph 9904272 v2;
I. Antoniadis, B. Pioline, Large dimensions and string physics at a TeV, LANL hep-ph 9906480;
T.Banks, M.Dine, A.E.Nelson, Constraints on the theories with large extra dimensions, LANL hep-th 9903019, JHEP 9906 (1999) 014.

[2] See, for example, J.Polchinski, Quantum gravity at the Planck length, LANL hep-th 9812104

[3] P.Horava, E.Witten, Nucl.Phys.B 475 (1996) 94-114;
M.J.Duff, Scientific American, February 1998, p.54-59.

[4]See, for example, F. Scardigli, Phys.Lett.B 452 (1999) 39-44

[5]See, for example, L.D.Landau, E.M.Lifshitz, Quantum Mechanics, Pergamon Press.